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Proper Equation of Fracture Curve

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This article contains a presentation of the proper equation of fracture curve and a treatment of requirements with which this equation has to comply. It has been proved that using an exponential curve for this purpose is erroneous. Studies have been presented and discussed, which show that the curve of fracture is a function of two variables. In this article, a proposal of mathematical equations is made, which exposes the curve of fracture as a function of two variables describing the state of stress. Practical applications of this theory have also been displayed.

Keywords deformability, fracture curve, ductile materials, state of stress

1. Introduction

In the metal forming industry, it is important to know how to calculate the permissible deformations in processes such as drawing, particularly tubes, or rolling by rolling mills. This is also to be done at low costs if possible, without making many tests during startup of the production. Therefore, it is necessary to define the deformability properties (plastic ones) of steel and to determine the calculation modeling of permissible deformations within the process, *e.g.*, of tube drawing on the basis of strength tests such as tensile, upsetting, or torsion tests.

It also should be kept in mind that during the process, *e.g.*, of cold rolling the strips, large drafts may be applied in roll passes and the strip will not undergo fracture. However, this is not being done for technological reasons, because high drafts rolling reductions cause deflections of rolling rolls that are too strong, and the strip will not have adequate cross-sectional shape. Therefore, in addition to the limitation in permissible deformation of materials caused by their deformability, we also have technological limitations resulting from the necessity to obtain a respective shape of the product or because of small tolerances.

2. Equation of Curve of Fracture as a Function of One Variable

The deformability of materials is a function of two variables: material and stress state. A classical experiment in this line was performed in 1912 by Karman.[1] He succeeded in obtaining large plastic deformation of brittle materials such as marble and red sand stone under conditions of high hydrostatic pressure acting onto the side surfaces of specimens when subject to deformation. Consequently, materials that undergo fracture without traces of plastic deformation, *e.g.*, during tension test, may be plastically and permanently deformed under application of tests using large hydrostatic pressures.

For quantitative determination of steel susceptibility for plastic cold processing, two proceeding methods are usually applied. In the first one, on the basis of operational practice, some indexes are fixed, making precise the deformability in a given process. For example, in Ref 2, it is mentioned that the ratio R_e/R_m and Z, where R_e = yield stress in uniaxial tension test, R_m = tensile strength, and Z = contraction, can be considered as evaluation criteria of steel ability for plastic deformation. It is stated in Ref 2 that $R_e/R_m = 0.50$ to 0.65 and $Z > 50\%$ ensure optimum susceptibility of steel for cold upsetting.

In the second method, full use is made of the fact that for quantification of deformation dependence in the stress function, the notion of the curve of fracture, also frequently called the curve of deformability or the curve of boundary deformability, has been introduced. This allows for modeling of plastic treatment processes by means of strength tests or reception tests or by forming theoretical deformability models. This also permits the determination of the steel susceptibility for plastic deformation.

The curve of fracture is presented in the coordinate system y = quantity describing the deformation and x = quantity characterizing the stress state factor, which is also called the stress triaxiality. Whereas the dependent variable considered are quantities varying slightly in final effect, as are the independent variable, the ratio $k = (\sigma_m/T)$, where σ_m = mean stress and T = intensity of shearing stresses, is accepted.

On the grounds of theoretical reasoning, Schiller[3] has derived the equation of fracture curve, which has also been very precisely treated in the survey paper.[4] It has the following form:[3]

$$\varphi_2 = \varphi_1 \left(\frac{\sigma_m}{\sigma} + \frac{2}{3}\right)^{-\frac{1}{n+1}} \qquad \text{(Eq 1)}$$

where

σ_m = mean stress,
σ = effective stress,
n = strain-hardening exponent in strain hardening curve,
φ_1 = deformation determinated in the test of uniaxial tension, and
φ_2 = deformation in the given test.

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Because φ_1 and n are determined in the uniaxial tension test, Eq. 1 means that, on the basis of this test, the equation of the entire curve of fracture may be determined. Verifying the examinations in Ref 3 with four materials and for various stress states has proven the correctness of Eq. 1.

Grosman[5,6,7] obtained an identical equation, which determines the curve of fracture. His reasoning was as follows: the curve of fracture must be described by means of a continuous curve having two asymptotes. One of them will be the *x*-axis. This is due to the fact that shearing stresses are necessary for creating plastic deformations. For triaxial uniform tension, according to the hypothesis of maximum shearing stresses and the hypothesis of distortion plastic work, shearing stresses equal zero; in other words, plastic deformation is not obtained. For this case, the quantity describing the deformation will be equal to zero and the stress state factor will tend toward infinity, because the denominator-intensity of shearing stresses will be zero. The second asymptote will be the straight line parallel to the *y*-axis and situated on the side of negative values of the stress state factor. Grosman has based the assumption of the existence of this asymptote on the investigations of Erbel,[8] who has proved the existence of the so-called bonding pressure at which obtaining of any broad plastic deformation is possible.

Out of many functions having two asymptotes, vertical and horizontal, Grosman[5,6,7] selected the power curve with the following form:

$$\bar{\varepsilon} = \frac{a}{(k - b)^c} \qquad \text{(Eq 2)}$$

where

$\bar{\varepsilon}$ = effective strain at fracture;
k = stress state factor, also called stress triaxiality; and
a, b, c = material constants.

From the constants appearing in the formula in Eq 2 c is always positive, whereas $b = -2/3$ was taken by Grosman on the basis of Eq 1, although in fact, $b = k_s = -p_s/T$, where p_s means bonding pressure.

To prove his hypothesis, Grosman[5,6,7] assigned the curve of fracture by means of the following tests: torsion tests of cylindrical specimens, tension tests of cylindrical ones, and tension tests of cylindrical ones with bored notches and with various values of diameter ratio of the smallest cross section to curvature radius. Really, two kinds of specimens were used with $d_0/R_0 = 2$ and $d_0/R_0 = 4$. Because a relation is considered to exist between a materials ability to strain harden and its deformability, Cu 99, 9 E, Armco iron E, and steel OH18N9 were used in the tests; these materials have considerably different values of the strain-hardening exponent n. To evaluate the approximate correctness of the measuring data obtained by means of the power function, Eq. 2, Grosman[5,6,7] used the F (Fisher) test, thus confirming his hypothesis.

Due to the importance of the existence of the vertical asymptote, the elaboration of Erbel is worth comment.[8] The objective of this paper was to determine the strain-hardening curves for

Table 1 Values of stress state factor $k = (\sigma_m/T)$ and Lode's Factor μ_6 for unlimited deformability of some alloys[9]

	k					
Alloy	D16	WD1	AD31	AMC	AD1	Ł96
$\mu_6 = -1$	−2.3	−2.0	−0.7	−1.7	−1.0	−1.9
$\mu_6 = 0$	−2.2	−1.3	−0.7	−1.0	−0.6	−1.5

very large deformations. The torsion test of ring specimens was applied in special equipment, which prevented the material to flow out, which enabled the triaxial state of stress to closely approach the hydrostatic compression. Researchers have proven that microvoids may get bonded if the material deformation is effected at sufficiently large hydrostatic pressure. The minimum value of pressure at which the material in a macroscopic scale does not lose cohesion is a characteristic quantity for each material, and it is called "bonding pressure," p_s. In Ref 8, this pressure was determined for copper M1 $p_s = 5000$ kg/cm^2, 490 N/m^2 and for aluminum A10 $p_s = 1870$ kg/cm^2, 183.26 N/m^2.

Torsion was operated at pressures of $p > p_s$. Therefore, there existed conditions ensuring obtainment of any great deformations, because the angle of torsion was not limited. Torsion diagrams[8] looked alike: at first, the moment increased, and afterward, it established itself on a defined level. These deformations are called "great deformations," and at the torsion of the ring specimens of aluminum, they begin at the deformation of $\varphi_i > 20$. In Ref 8, this deformation was recalculated onto contraction, obtaining $Z = 99.99999984\%$, which shows that, in this test, it is possible to obtain very large deformations.

In Ref 9, are given a boundary value of the stress state factor of $k = \sigma_m/T$ and a value of the Lode's factor of $\mu_\sigma = \frac{2\sigma_2 - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3}$, at which, experimentally, an infinite deformability was obtained for several aluminum and copper alloys; these values are presented in Table 1. When analyzing these data, it may be found that these values are not large and that the vertical asymptote is relatively near the dependent variable axis.

One may imagine as well that in the case of realizing the triaxial uniform compression, we will receive an unlimited deformability, because the material simply will not know where to flow and will "resist" more and more strongly. In such a situation, not only will not cracks appear but one may imagine also that the supplied energy will be so large that it will cause recrystalization, as is the case during hot rolling.

In the opinion of the present author, the results of these experimental investigations and the aforementioned theoretical reasoning serve as sufficient evidence for the existence of the vertical asymptote, where, for $k \rightarrow k_s$, $\bar{\varepsilon}, = f(k) \rightarrow \infty$.

As far as the existence of horizontal asymptote is concerned, which is the axis of the independent variable of the state of stress factor, I did not discover any research indicating that anyone had succeeded in realizing, in practice, a state of triaxial, uniform tension in which the plastic material would undergo brittle fracture. This does not mean that this assumption should be rejected. However, it is worthwhile to draw attention to another aspect of the matter. In Ref 10, a method for determining the distributive strength, brittle cracking strength was described.

The distributive strength ratio to the yield point R_0/R_e for several metals was also calculated. According to Orowan,[10] the highest value of this ratio, at which a sharp notch or cracking may still cause brittle cracking of a material, is from 2.6 to 3.3, and this range includes steels and armco iron, for which the brittle cracking strength was determined in Ref 10.

Meanwhile, the value received for copper is $R_0R_e = 9.2$, which indicates its high deformability and plasticity. The sharpest notch cut on a tensioned specimen will not create conditions for evoking in it the brittle cracking found with normal temperature and at normal load speed.[10]

Results of these investigations clearly show how difficult it will be to develop a test in which it might be possible to obtain brittle fracture of materials having very large deformability, such as copper at ambient temperature and at static speed of deformation. However, this does not change the fact that, from a theoretical point of view, the *x*-axis is the asymptote of the curve of fracture.

However, in the literature, an erroneous understanding of the mathematical substance of the fracture curve exists. For its elaboration, advantage is being taken of the known Rice–Tracey equation:[11]

$$dR/R = 0.28\, d\bar{\varepsilon}\, \exp\,(3\sigma_m/2\bar{\sigma}) \qquad \text{(Eq 3)}$$

describing the growth of an initially spherical hole in a rigid, nonhardening matrix.

On the basis of this equation, Hancock and Mackenzie[12] have obtained the largely known equation for failure strain in the form of

$$\bar{\varepsilon} = \varepsilon_n + \alpha \exp\,(-3\sigma_m/2\bar{\sigma}) \qquad \text{(Eq 4)}$$

where ε_n is the void nucleation strain and α is the material constant. The existence of ε_n is explained by the fact that there are many materials in which appreciable plastic flow occurs before voids nucleate.

Meanwhile, in light of what has been said above, this is untrue, because two quite different processes are being discussed here. In the process of void nucleation, growth, and coalescence, one may assume the existence of ε_n, if there are no brittle particles in the material, which would crack at the beginning of deformation. However, in the curve of fracture for which the *x*-axis is an asymptote, there is no place for a parameter of this type.

Another characteristic mark for the process of void nucleation, growth, and coalescence is a very rapid increase of deformation for high values of $\sigma_m/\bar{\sigma}$. This process starts slowly at the beginning of void nucleation and further follows slow growth and nucleation until, at a certain point, it begins to violently speed up; what follows then is void coalescence and catastrophe, in other words, fracture of the specimen. The exponential curve mostly fits the description of such a catastrophic process. Let us take, for instance, the function of $y = 10^x$. For $x = 1$, $y = 10$ and for $x = 3$, $y = 1000$, or otherwise whenever the independent variable increased 3 times from 1 to 3, the dependent variable increased from the value of $y = 10$ to $y = 1000$, or 100 times. Therefore, curves of this type were willingly used in reports of the Roman Club, where research was conducted into the effects of an increase in world population and

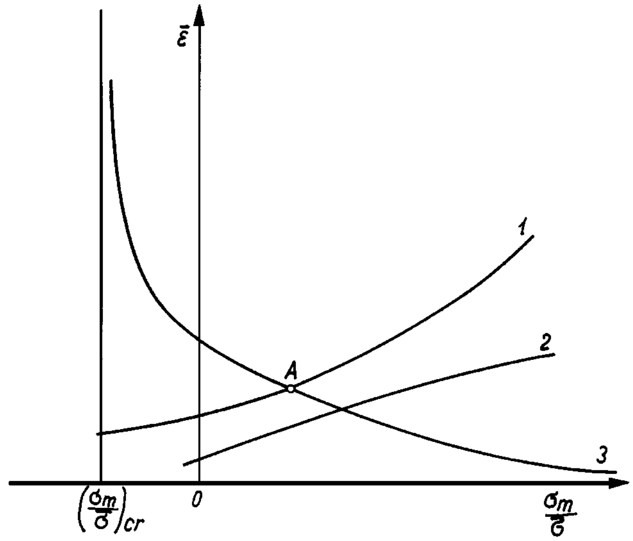

Fig. 1 Curves of fracture and void growth

the exhaustion of raw materials; exponential curves were used in the description of these catastrophic processes.

Meanwhile, in reality, the curve of fracture has two asymptotes and the existence of the vertical one is caused by the fact that, under conditions of great hydrostatic pressure, the phenomenon of unlimited deformability arises, caused by bonding of cracks. This is not only theory, because, in practice, it happened to succeed in performing this event, as mentioned before.

Figure 1 helps provide a better understanding. Curves 1 and 2 are curves of void growth; it seems that curve 1 better describes this phenomenon, because it grows faster in the final stage of deformation. These are curves describing a catastrophic process, always speeding up, that is, the fracture of a specimen under deformation. This process is described quite well by the exponential curve. Fracture of a specimen is done at point A, and the curve on which such a point A is placed being received for various specimens, thereby for various states of stress, is the curve of fracture. For a description of such a curve, a power function is required, which has two asymptotes indispensable for mathematical description of two effects, the unlimited deformability and the brittle fracture of plastic materials. A description of the constantly speeding up, catastrophic increase is not required for this curve; therefore, it should have another mathematical form or equation.

It seems to me that in the literature another mistake has been made as well. For the description of curve 1 in Fig. 1, an increasing curve of type $\bar{\varepsilon} = a \exp\,(\sigma_m/\bar{\sigma})$ should be used, whereas if the same function were applied for a description of the decreasing curve 3 (curve of fracture), it should have the form $\bar{\varepsilon} = a \exp\,(-\sigma_m/\bar{\sigma})$; in other words, a change of symbol should take place.

The question arises: Why has such a flawed attitude been maintained so long. The answer is simple. Only a few researchers have studied the process of unlimited deformability. Practically, the investigations are limited to performing tensile tests of cylindrical specimens with notches of various radii and those specimens without notches and to performing torsion tests of cylindrical specimens, with longitudinal tension sometimes added. The results obtained usually contain a small range of stress state factors, and for the approximation of results obtained, many functions may be used, sometimes even parabola or straight line, thus receiving good results.

Here, some publications bring to light doubts. In Ref 13, in Fig. 3, results of investigations for Swedish Iron are presented, and this figure has the character of Fig. 1 from the present publication. The curve of fracture has a correct decreasing course, and curves describing deformation histories for axisymmetric notch specimens are increasing, which is correct as well. On the other hand, doubts are coming to light concerning the curves describing deformation histories for plain strain notch specimens and for plain strain specimens, which are straight lines, which suggests the existence of a constant factor of stress state during the entire deformation process. I do not believe this would be possible; rather, I believe that a measuring error is more likely its effect. Otherwise, this finding would mean that the dream of every researcher was discovered, i.e., a specimen with a constant factor of the stress state.

In Ref 14 in Fig. 6 and 7, which have the same character as Fig. 1 of this article, results have been presented of research for notched specimens, with different notch radii of curvature, deformed in tensile test. The curves of fracture have a correct, decreasing character, although their mathematical equations are not correct power curves. However, from the curves describing the process of void growth, only one has the character of curve 2 from Fig. 1 of this article. The rest of the curves are a bundle of decreasing curves "parallel" to the curve of fracture, which brings to light some doubts, because it seems that in the said deformation process, the increase of deformation should be accompanied by an increase of the stress state factor in the direction of its positive values, because these are tensile stresses, which are causing fracture of the specimen.

In Ref 9, for a description of cold-obtained fracture curves in various tests, the least-squares method has been used. An exponential curve of the following form was selected:

$$\Lambda_p = a \exp (b\, \sigma_m / T) \qquad \text{(Eq 5)}$$

where Λ_p = slip deformation degree, whereas a and b are constant factors for the material given; factor b is always negative. These curves are always determined for $\mu_\sigma = -1$ (tension) and $\mu_\sigma = 0$ (torsion). In other words, two curves are always being determined. It should be mentioned as well that the Lode's factor in the tension test is independent from the notch size and from the quantity of hydrostatic pressure, when making the tension test under conditions of hydrostatic pressure, and it is constant during performance of the test.

In Ref 9, no physical justification was given for selection of the exponential curve [Eq. 5] for approximation of experimental data to obtain the equation of fracture curve. It simply seems that one of the computer standard programs was selected, for approximation of the obtained experimental data. From a theoretical point of view, this is an erroneous assumption, because a proper equation describing the curve of fracture should have two asymptotes, which may include a power curve, and Eq 5 does not fulfill this condition.

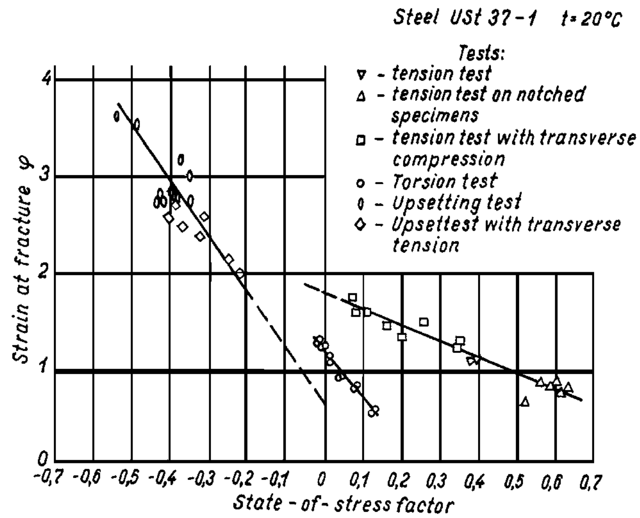

Fig. 2 Curve of fracture for Ust 37-1 steel[15]

3. Equation of Fracture Curve as a Function of Two Variables

In the Vater and Lienhart paper,[15] being a continuation of the Stenger elaboration,[16] results have been cited of deformation studies in the function of stress state for two steels, different temperatures, and deformation velocities. For determination of the fracture curve, the following tests were used: upsetting, upsetting with transversal tension, torsion, torsion with longitudinal tension, tension of plain specimens, tension of specimens with notch, and tension with transversal compression. Figure 2 shows a typical fracture curve obtained for USt37-1 (0.16%c) steel in room temperature. When analyzing the data shown in Fig. 2, it may be found that points relating to a given test, *e.g.*, upsetting, are placed on one straight line. The positioning of a point on this line defines the size of the variable diameter in the given test; *e.g.*, in the torsion test with longitudinal tension, this is the lengthwise axial force quantity. In the treated elaboration, the appearance of several lines in the diagram is to be explained by the effect of stress σ_2, according to the argumentation of Stenger.[16] It means in this approach that the deformability of metals depends not only upon the state of stress but also upon the position of the middle principal stress σ_2 in relation to both remaining principal stresses. At equal values of the stress state factor, the deformability is larger the lesser is the difference between the middle principal stress σ_2 and the smallest principal stress σ_3. This has been presented as a diagram in Fig. 3.[16]

In Ref 17 are presented the results of deformability studies of several materials under application of tension and torsion tests under conditions of high hydraulic pressure, acting onto lateral surfaces of specimens being deformed. Results of the studies are shown in Fig. 4. It should be remembered that the Lode's factor takes the value of $\mu_\sigma = -1$ at tension, $\mu_\sigma = 0$ at torsion, and $\mu_\sigma = +1$ at compression. Fracture curves were obtained by approximation of experimental data, with the method of least squares, by means of the parabola with the following equation:

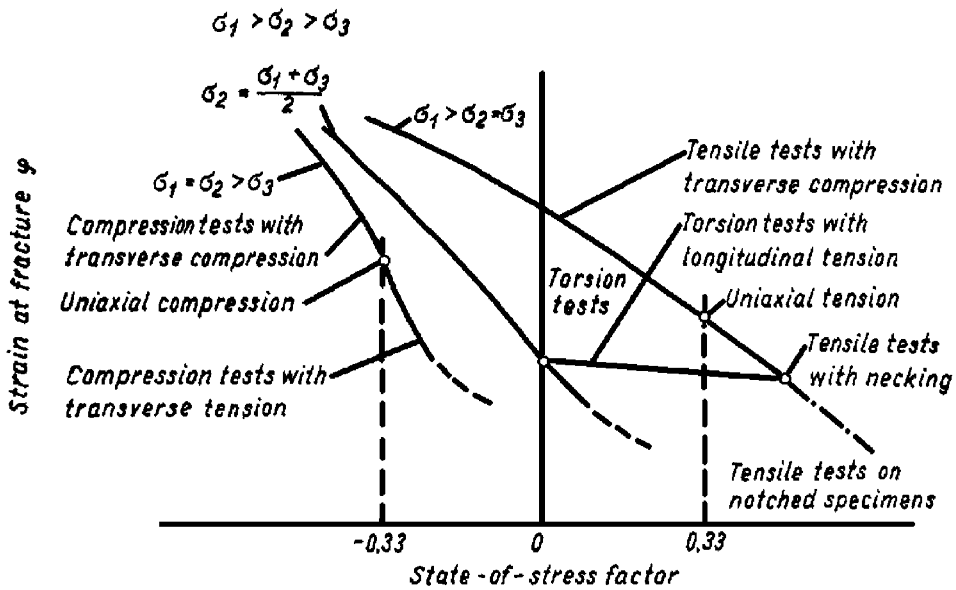

Fig. 3 Effect of mean stress on curve of fracture[16]

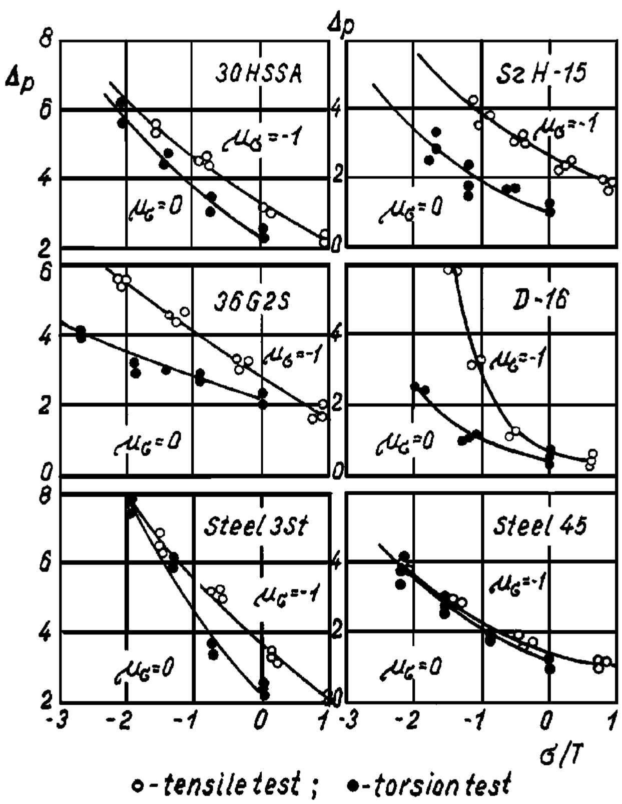

Fig. 4 Curve of fracture[17]

$$\wedge p = ak^2 + bk + c \qquad \text{(Eq 6)}$$

where $k = (\sigma_m/T)$ = state of stress factor.

When analyzing the data presented in Fig. 4, it should be noticed that, for all tested materials, the curves obtained during the tension test under conditions of hydrostatic compression

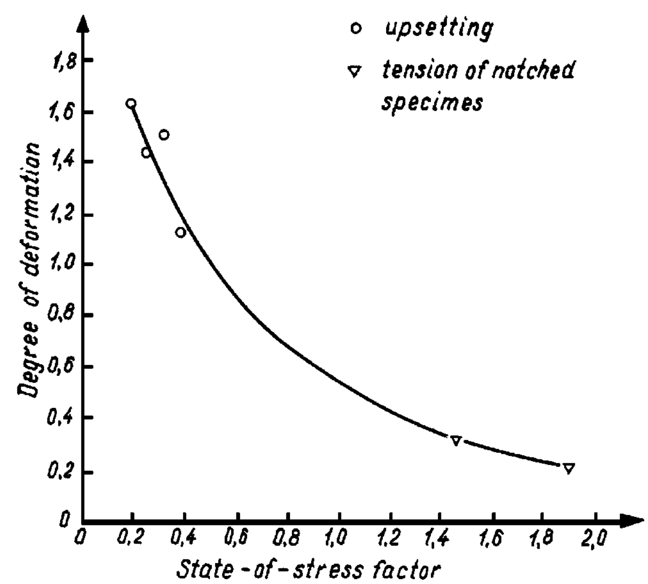

Fig. 5 Curve of fracture for K18 steel[18,19]

$\mu_\sigma = -1$, are situated above the curves obtained at torsion under conditions of hydrostatic torsion, $\mu_\sigma = 0$. It also can be seen that with an increase of hydrostatic pressure, *i.e.*, with a decrease of k, irrespective of the type of test, the plasticity increases and its intensity depends upon the material and μ_σ. A curious observation is also the fact that results obtained in one test are set up along one line.

The author of the present paper has determined the fracture curve of K18 steel, which has, among other elements, 0.19% C and 0.98% Mn.[18,19] To determine the curve presented in Fig. 5, the tensile test of specimens with notch values/ $d_0/R_0 = 0.5$ and $d_0/R_0 = 4$ and the upsetting test of cylindrical specimens with $h_0/d_0 = 1$ under utilization of various lubricants have been applied. Figure 5 shows that points obtained in the upsetting test are situated on one straight line.

When analyzing the curves of fracture, as shown in Fig. 2 to 5, it should be noted that the results of upsetting tests, with various *h*/*d* ratios and various lubrications; torsion tests, without and with participation of hydrostatic pressure and without and with longitudinal tension; and tension tests, of flat specimens and with notches of different curvature radii *R*, without and with participation of hydrostatic pressure, are situated along certain lines that form a family or a bundle of curves. In a simpler case (Fig. 5), the straight lines obtained in the tests of upsetting and tension of specimens with notch form arms of the fracture curve. In more complicated cases, however, these are already independent curves. Therefore, it may be said that after having done exact and numerous examinations of fracture curves under participation of hydrostatic pressures, instead of one fracture curve, we have a family or bundle. Stenger[16] explains the existing dispersion of results by the effect of stress σ_2, where $\sigma_1 > \sigma_2; > \sigma_3$, but in Ref 9 and 17 the results are explained by the effect of Lode's factor. This means that the fracture curve is a function of two variables: the stress state factor and a second factor also describing the state of stress.

In the present paper, proposals of mathematical equations defining the fracture curve as a function of two variables are shown. Both variables independently describe the state of stress. In the first of them, the stress state factor was accepted, a long-time frequent practice in the literature. The second variable is proposed to accept the factor σ_1/σ, which presents the ratio of maximum tensile stress to effective stress, which causes this factor to be independent of of the kind of material used. This is caused by the fact that this stress occurs in every test accepting the value > 0, and together with the hydrostatic pressure, factor *k*, these factors are the most important ones determining the fracture of material factors. In this connection, it is proposed that the equation describing the curve of fracture as a function of two variables in the following form be accepted:

$$\bar{\varepsilon} = \frac{a_1 - a_2 \dfrac{\sigma_1}{\sigma}}{(k - b)^c} \qquad \text{(Eq 7)}$$

The second proposal is to accept, instead of factor σ_1/σ, the Lode's factor in accordance with Ref 9 and 17 as a second independent variable describing the state of stress. In this case, the equation of fracture curve as a function of two variables will take the following form:

$$\bar{\varepsilon} = \frac{A_1 - A_2 \mu_\sigma}{(k - B)^C} \qquad \text{(Eq 8)}$$

where a_1, a_2, b, c and A_1, A_2, B, C are coefficients to be determined on the basis of experimental data; c and C are always positive. Of course, these coefficients are course material constants.

Equations 7 and 8 in the following form have not been accepted:

$$\bar{\varepsilon} = \frac{1}{\left(a_1 - a_2 \dfrac{\sigma_1}{\sigma_1}\right)^{c_1} (k - b)^c} \qquad \text{(Eq 9)}$$

$$\bar{\varepsilon} = \frac{1}{(A_1 - A_2\, \mu_\sigma)^{C_1}\, (k - B)^C} \qquad \text{(Eq 10)}$$

because introduction of coefficients c_1 and C_1 would complicate the equations too much. On the other hand, it should be kept in mind that only one vertical asymptote exists, connected with the fact that, if $k = k_s$, then, in the test, bonding pressure will be obtained and, in this connection, an unlimited plasticity. However, there are no such boundary values for coefficients $\sigma_1/\bar{\sigma}$ and μ_σ, which is illustrated best by the data presented in Table 1 and Fig. 4. Therefore, these elements of the equations cannot be placed in the denominator. Figure 4 shows that the factor μ_σ is constant, *e.g.*, in the tensile test, namely, for various types of specimens and different kinds of testing. The important elaboration in Ref 20 also should be remembered, where it is ascertained that, for a decrease of the volume of cracks, the joint action of hydrostatic pressure and the plastic deformation is necessary. This is caused by technical difficulties, which do not permit the use of the huge pressures required "to cure" the cracks, without the simultaneous action of plastic deformation. However, both Eq 7 and 8 have a horizontal asymptote for the case of uniform triaxial tension, while for $k \to \alpha$, $\bar{\varepsilon} \to 0$.

Trials were made to give a mathematical form to the reasoning of Stenger[16] concerning the existence of several fracture curves, which was previously described in writing. For this reason, the possibility of accepting the following factor describing the state of stress was considered:

$$k_1 = \frac{\sigma_2 - \sigma_3}{\sigma_2} \qquad \text{(Eq 11)}$$

However, in the case where $\sigma_2 > 0$ and $\sigma_3 < 0$, an accumulation of values of tensile and compression will follow, which is not right when considering the fracture of material. However, if in the numerator of Eq 11 the symbol $-$ is replaced by the symbol $+$, then we receive an index that is an incomplete state of stress factor, which seems incorrect.

In this paper, the present author has chosen to discuss the factor describing the state of stress, which could replace the factor $\sigma_1/\bar{\sigma}$ in Eq 7. This is done, therefore, because now it does not seem to be possible to obtain the equation of fracture curve as a function of two variables, in a theoretical way. For this reason, only approximation of the experimental data remains and, for this purpose, the verification of some curves. The best accuracy of approximation should be determining the selection of the function. In the opinion of the present author, the best results should make use of the factor $\sigma_1/\bar{\sigma}$, because it includes the maximum tensile stress, and this is the quantity without which the fracture of material will not follow.

4. Correlation of Boundary Deformations

As a result of the fact that the results of various tests are setting along the fracture curves, the conclusion that there may be some correlations between boundary deformations of various tests has been drawn. Such tests were performed for several carbon steels after different heat treatments, the results of which have been presented in Ref 21 and justification of the results in Ref 4, 22, and 23. On this basis, the theory of correlation of boundary deformations from various tests has been erected.[21] The performed experimental investigations have confirmed this theory and have revealed[21] that, by means of torsion of cylindrical specimens, the modeling of boundary deformations from tensile tests and upsetting the cylindrical specimens is possible. The effected investigations have proved as well that, by means of tensile tests of cylindrical specimens, the possibility of boundary deformation modeling from the upsetting test of cylindrical specimens exists. It has been proved as well that, by means of the impact resistance numerical value from the Mesnager test, the modeling of boundary deformations from tensile and upsetting tests of cylindrical specimens is possible.[21]

Recently, verification concerning this theory was made on specimens of other geometries, cut from some carbon steels, after different conditions of heat treatment. This verification has entirely confirmed the theory to be the right one. A respective paper is now under preparation for publication.

An important conclusion from the theory about the correlation of boundary deformations and from the fact that fracture curves exist is the possibility of standardization of specimens. This will enable mutual utilization of results of investigations and will reduce their costs. However, this is a matter to which no due importance is being attached, as I see it.

5. Discussion on Results

The process of growth of voids, as shown in Fig. 1 by means of curve 1, has been the subject of very intense study for many years. This study describes utilized, increasing exponential curves, which is right, because they are appropriate for a description of more and more speed, until a final catastrophic process. In these equations, often, void nucleation strain is taken under consideration as well, and it should be if, in the material, there are no brittle particles, which crack just after starting the deformation. When watching such a process of escalation of plastic deformation, as shown in Fig. 1 by means of curve 1, we finally come to point A, where plastic fracture of the material begins. It is differently defined in the literature, but this is of no importance in the present discussion. Point A present fracture in the given stress state is also situated on the fracture curve (curve 3 in Fig. 1). And here begins the problem that is not understood in the literature. Other laws are governing the curve of fracture: on one side, the existence of unlimited deformability; and on the other side, the lack of plastic deformation at the uniform triaxial tension test. This means the existence of two asymptotes, and the curve should be a power curve. It should be kept in mind[20] that for "curing" or for bonding of voids, the joint action of hydrostatic pressure and plastic deformation is required. Until now, nobody was succeeding at performing the huge pressures required for curing (bonding) the voids. This means that other principles rule the fracture curve, and therefore, it should have another character (form), and this is not understandable. Precise studies undertaken with the use of tests performed with an application of big hydrostatic pressures also show that the fracture curve is a function of two variables describing the state of stress, which is also generally not understandable.

Figure 4 in Ref 24 and Fig. 6 and 7 and 13 to 15 in Ref 25 reflect the misunderstandings connected with this matter. In the present author's opinion, only one point is correct within them, that is, the fact that these are decreasing curves, and the remaining marks on these curves appear doubtful. They are the following:

- assuming as a horizontal asymptote a more closely undefined line parallel to the *x*-axis, and this should be the *x*-axis;
- lack of vertical asymptote existence or mention of it;
- use of something other than the power function for function approximation;
- lack of information in the literature review regarding opinions that the curve of fracture is a function of two variables and presented elaborations based only on limited investigations; and
- using only smooth and notched tensile specimens to appoint the fracture curve; it is in these tests that the most disadvantageous state of triaxial stresses and fracture quickly arises.

It seems a bit as a trial to evaluate the life of a 70-year-old man on the basis of the first 10 years of his life. Also, the range of results obtained is relatively narrow, while there are many curves appropriate for their approximation. This allows the choice of a rather accidental approximating curve, which makes subsequent understanding of the substance of the matter difficult. Another problem is the fact that notched tensile specimens do not correlate with other tests,[4,21] and, therefore, in this author's opinion, trials to generalize such results cause much doubts.

I hope that the authors of the articles regarding which I express my doubts will understand that I am doing so in an impersonal way. However, if researchers continue to develop some of these opinions, which I personally consider erroneous, doing so will cause serious losses. For this reason, I think that this matter deserves serious discussion. If this is not done, many researchers may loose time by developing erroneous theories.

In connection to this, I propose a threefold approach to the problem.

- For strict scientific purposes, the existence of a fracture curve as the function of two variables should be assumed. However, it should be remembered that their determination by approximation of experimental data may be unusually expensive. In elaboration,[9] investigations were made using the following tests: tension of cylindrical specimens smooth and with notches, under hydrostatic pressure and without it; tension of flat specimens without notches and with notches; torsion of cylindrical specimens smooth and with notches, under hydrostatic pressure and without it; torsion with axial load under hydrostatic pressure and without it; and upsetting of cylindrical specimens with various ratios of height to diameter and under use of various lubricants as well as pressure tests. Yet, to date, the fracture curve equation as a function of two variables by approximation of the experimental data has not been worked out. This illustrates well the range of difficulty and expense.

- For practical scientific purposes, the fracture curve equation as a function of one variable, *i.e.*, of the stress state factor in the form of a power curve, should be accepted. For its determination, it should be enough to make torsion tests of cylindrical specimens, tensile tests of cylindrical specimen and cylindrical specimens with notches, and upsetting tests of cylindrical specimens. It should be remembered that, in the upsetting test of cylindrical specimens as well as in the bend test of flat specimens for more plastic materials, the fracture may not be obtained.
- For industrial purposes, we should obtain the data set *via* the use of the most simple approximations and with the aid of the best approximating functions. In Ref 26, for approximation of fracture curves of two kinds of zinc, the straight line was used with good effect. Sometimes this can be a parabola. Other curves may also be used. Utilization of fracture curve modeling the tube drawing process has been described in Ref 27. I participated in this operation; therefore, I can confirm that, because of this theory, there has been success in introducing new tube drawing theories, and in intensifying the production and reducing the number of interoperational annealings. In Ref 28, through extrapolation of experimentaly obtained fracture curves up to intersection with the *x*-axis, values were obtained of the stress state factor at the transition from plastic fracture to the brittle one. They are for zinc (Zn)/$(\sigma_m/T) = -0.4$ and for beryl (Be) $\sigma_m/T = +0.5$. This method can be regarded as a practical determination test of brittle fracture for plastic metals.

Attention should be drawn as well to the fact that, if we consider the fracture curve as a function of two variables, then the vertical asymptote should appear only for one variable, namely, for the stress state factor. This complies with the achievement in the test of bonding pressure and unlimited deformability. However, there are no such boundary values for Lode's factor (Fig. 4 and Table 1), which assumes a constant value in tensile tests of specimens of various types. The same occurs for the torsion test in the described range. Also, the factor σ_1/σ does not reach the boundary value, which is clear because it represents the maximum tensile stress, which is contrary to unlimited plasticity. This is an interesting observation.

It should also be realized that I did not meet the results of investigations concerning unlimited deformability for fracture curve needs. The target of the very interesting investigation in Ref 8 was to define the flow curves for large deformations, and what is most interesting for the requirements of this article is the subordinate effect of these investigations. The same has appeared in Ref 9, where the entire method was developed out of determination of boundary deformability, with particular attention given to its practical application for various processes of plastic treatment. The book in Ref 9 has 140 pages, but only 9 lines have been dedicated to unlimited deformability, 3 lines of which present data shown in Table 1 of this article. Therefore, one should be aware of the fact that the problem of the vertical asymptote and of the bonding pressure is still open and awaits the researcher who would solve it. In this situation, only the theoretical and logical thinking remains for us, with the purpose of establishing the principles ruling the curve of fracture.

In Ref 29 an interesting story is shared from the life of Isaac Newton, who, in his life, among other things, when sitting at his desk, determined the precise position of the planets on the basis of the law of universal gravitation. Afterward, he compared his calculations with the results that were sent to him from Greenwich by John Flamsteed, the founder and director of the famous observatory who had at his disposal the best, at this time, measuring apparatus. But Newton, on the basis of his own calculations, called into question Flamsteed's results and pointed out where the measuring error must have been. After verification, it came out that the results Newton had determined while sitting at his desk were right. There are some people who say that Flamsteed broke (stopped) cooperation with Newton because of mathematical character of nature (that nature is mathematical—nature is governed by simple mathematical rules (laws)).

I am purposely citing this authentic story, which sounds today as more of an anecdote. Nature is ruled by the laws of mathematics, and we are not allowed here to commit any errors. Sometimes during a moment of reflection, thoughts of great meaning arise. These ideas are particularly needed now, when new views on some concepts are arising. And this is what I am asking the readers for: a moment of attention and reflection for every critical opinion in this domain, having direct practical application.

6. Conclusions

- This paper includes a criticism of the attempt to transfer exponential curves describing the growth of voids into the mathematical description of the curve of fracture.
- A justification has been given as to why the power function should be applied to the mathematical description of the curve of fracture.
- It has been proved on the basis of literature that the curve of fracture is a function of two variables describing the state of stress. This curve must have a horizontal and a vertical asymptote. It should be pointed out that both asymptotes appear only for one independent variable (stress state factor) and not for two independent variables, as it may seem to be.
- Two equations for mathematical description of the curve of fracture as a function of two variables have been proposed. The next step should be to obtain measuring data and their approximation by means of proposed curves.
- The practical possibilities of utilization of this theory have been presented. Attention should be drawn particularly to the possibility of standardizing the specimens used for deformability tests of metals and materials.

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